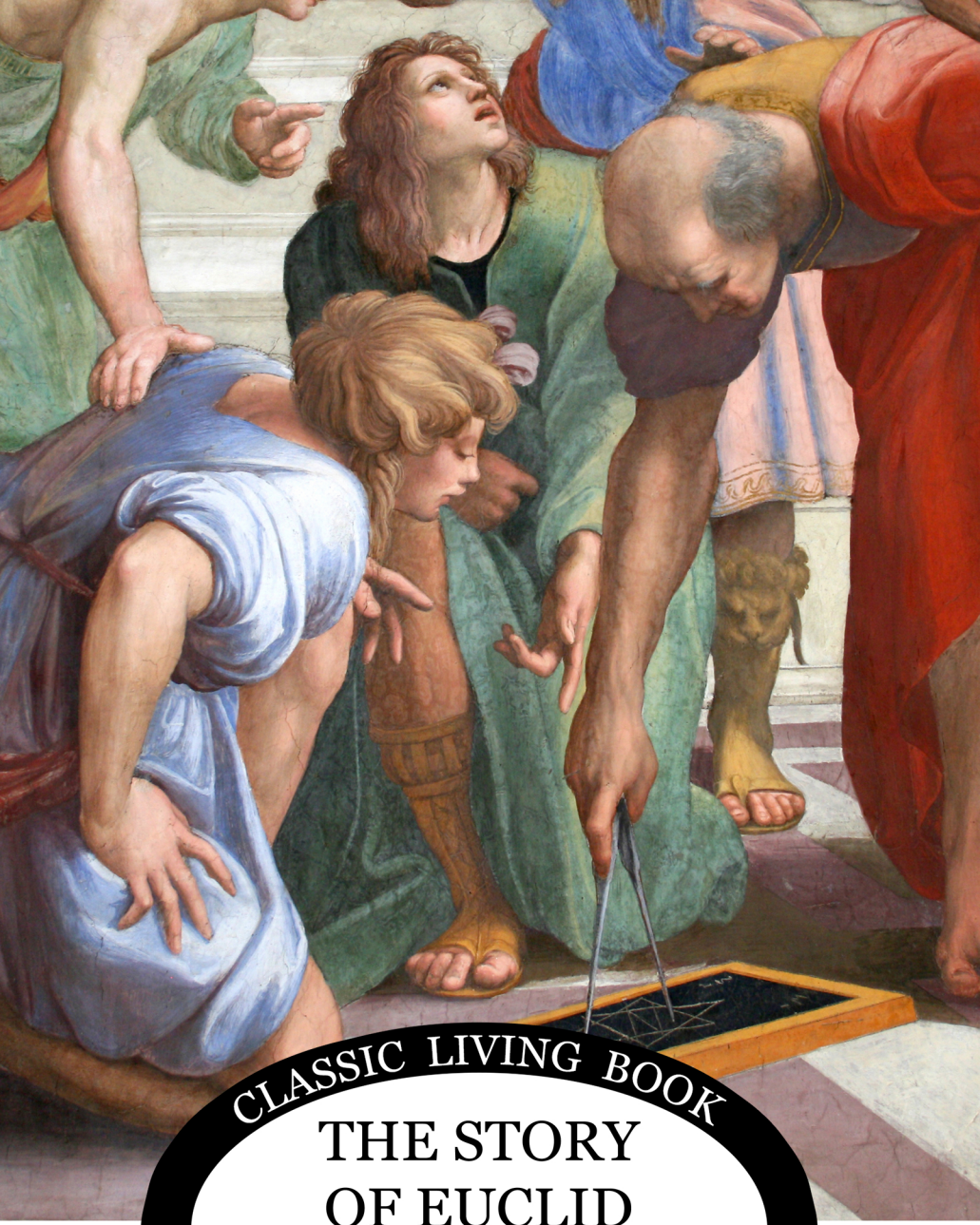

CLASSIC LIVING BOOK

THE STORY OF EUCLID

W. B. Frankland

COMPLETE AND UNABRIDGED

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The Story of Euclid

by

William Barrett Frankland

Glossary

To aid the modern reader, this edition of "The Story of Euclid" includes a glossary of terms. Some words may be unfamiliar due to their age or mathematical nature. This glossary aims to clarify their meaning, ensuring a smoother reading experience and a deeper understanding of Euclid's contributions to geometry.

appurtenances: Things that belong to something else as a less important part; additional features or equipment.

mensuration: The process of measuring, especially calculating geometric dimensions like lengths, areas, and volumes.

Megathological: Relating to deep contemplation of grand or lofty ideas, particularly those intended to elevate the mind towards the divine or abstract.

tiro: A beginner or a novice; someone new to learning a particular skill.

parallelopipeds: A solid 3D shape where all six faces are parallelograms (think of a slightly squashed box).

incommensurables: In geometry, this refers to lines or quantities whose lengths don't have a common divisor (or whose ratio cannot be expressed as a fraction of whole numbers). For example, the side of a square and its diagonal are incommensurable.

eleemosynary: Relating to charity or giving freely to those in need.

parallel-axiom: Also called Euclid's Fifth Postulate. It's a fundamental rule in geometry stating: If a line intersects two other straight lines, and the interior angles on the same side add up to less than 180 degrees, then the two straight lines will eventually meet if extended far enough.

Contents

EUCLID

Taken from a Brass Coin in the Repository of the late Queen Christian of Sweden

Euclid *the Mathematician was of* Alexandria, *where he taught in the Reign of* Ptolemy Lagus *in the* CXX Olympiad, *and Year of* Rome 454. *He Wrote many things relating to Musick and Geometry: But his* XV *Books of Elements (of which he is generaly thought to be only the Collector) are most applauded: the two last are attributed to* Hypsicles *of* Alexandria, *and not to him.* Cardan Vossius

PREFACE

THE career of the *Elements* of Euclid has been of an extraordinary and indeed unique character. For two thousand years or more it has served for text-book to countless students, and today it is a work with which all Englishmen are more or less familiar, whilst the name of its writer is a household word throughout the civilized world.

And yet so lacking in curiosity are we of this country, although I do not know that other people have shown themselves much more inquisitive in the matter, that there does not exist, so far as I am aware, any small book of this sort to tell us what in the Story of Euclid is of interest or importance for busy men and women to know. Even in school books, I venture to suggest, there is a quite unaccountable lack of that historical and philosophical information which it is the aim of this little volume in some degree to communicate, and so in writing this brief sketch of Euclidian lore I have had in mind younger as well as older people.

I hope that no one will disapprove of these efforts on the ground that a little knowledge is a dangerous thing; for, whilst I have fears that in points of detail this old-fashioned aphorism may apply only too aptly to myself, it will be a very keen disappointment to me if, in its broad outlines, this panoramic

story gives to the reader any essentially wrong impression of the past and present of Euclidian geometry.

Of course the subject labours under the difficulty that occasionally, in order to avoid an unprofitable and insipid vagueness, I have had to resort to downright technical language. Still, should anyone find himself bogged or bored by the abstractness of odd paragraphs, I trust he will see fit to skip the offending places, and return to them after gaining a bird's-eye view of the whole matter.

On the other hand, it may be that some will find it a pleasurable relief, after a hard spell of school work, to stroll in the fields of history, where we almost seem to breathe something of a larger and freer atmosphere, through which great men of past ages appear, moving with a grand and solemn dignity.

I cannot but add a word of thanks to those friends who have assisted me in the correction of proofs and in other ways, especially in the preparation of an index. The works of which I have made use will be found referred to in the course of the book.

W. B. F.

CLARE AND SELWYN COLLEGES,
CAMBRIDGE.
Michaelmas, 1901.

CHAPTER I

INTRODUCTION

Philosophy can bake no bread, but she can procure for us God, freedom, and immortality. - NOVALIS.

ANY one who picks up this little book may ask how the "Story of Euclid" comes to take its place in a "Library of Useful Stories." If his schooltime is past, he may put the question, "Of what use can it be to me to learn any more about a thing for which I never cared but little, and of which I remember not very much?" Or, if his acquaintance with Euclid is still fresh, he may inquire, "How can this story help me, busy as I am with this or that examination?" In fact, at the opening of a new century, time is short, and correspondingly valuable; and this in itself demands that cause be shown why the story of Euclid should be told or read.

It is scarcely credible that very many readers will ever find it possible to turn the contents of such a book as this to account so as to procure for themselves certain pounds, shillings, pence, and farthings sterling. In that sense this story is not at all likely to be useful, though it may still plead usefulness on the ground that it strives to furnish pleasures of a wholesome if quiet sort. Moreover, these pleasures help to invigorate and develop those faculties which brutes do not share; and they

may be said to abide rather than to flit, and to enrich rather than to impoverish.

On scientific grounds, further, the story of Euclid has high claims to usefulness. All the great inventions are inseparable from natural science\,: without it there could be none of those mechanical and electrical wonders, at which no one wonders—the steam-engine, the telegraph, the dynamo, and what not. But natural science depends, first and foremost, on measurement; and wherever measurement is made more and more precise, a branch of science is in a fair way to become more and more perfect. And geometry is the science of measurement, in that she is concerned with the how and the why of the rule and compasses, and especially with the art of drawing to scale—only to put forward the simplest service she renders. This story tells how ideas about measurement have changed in the course of time, and so establishes some claim to usefulness from a scientific standpoint.

Then, as already suggested, the story of Euclid introduces questions and problems, at once simple and profound, in which the mind may exercise itself to its full bent. And it seems as if in the complex life which has to be lived today, the mind could not be too strong. Men and women of today, workers in a social scheme, citizens of a vast empire, members of an intelligent humanity, cannot escape the ambition which spurs them on to realize their heritage. No surface knowledge will suffice for those who are "the roof and crown of things." The mind is ever searching for the purpose and inwardness of what it discerns. It may be that this story will prove useful by providing practice in accurate thought about abstract matters.

A very old tale serves well to illustrate the manner in which geometry plunges below the surface. In days before Euclid the plodding student encountered the unpleasant gibe that in proving that two sides of a triangle were together greater than the third, he stated no more than every donkey knew. The fact was well known, so it was derisively observed, to every donkey who walks straight to a bunch of hay. Nevertheless the student had a sufficiently good retort: "Yes, but the donkey does not know the reason why!"

And yet intellectual gymnastics will be far from constituting the chief object of concern; although will be encountered problems, solved and unsolved, which vexed men's minds for many successive generations, still the history of Euclid's great text-book will be the main theme, and the narration of the fates of one of the half-dozen supremely marvellous books the world has seen, through the varied course of twenty-two long centuries. During this prodigious stretch of time the *Elements* have been the very inspired Scripture of mathematics and natural science; and they are still in a position of accepted authority, despite the destructive efforts of criticism. If it is useful to know how the Bible was composed and handed down, as well as to be acquainted with its contents and their significance, it is also useful, in however infinitely less a degree, to learn the story of Euclid.

It might not have been guessed that the history of geometry would march with the history of religion no less than with the history of science, and yet this is the case. Before Christianity had dominated by her own scheme all the highest culture of the world, the subtle restless mind of the pagan Greek strove

ceaselessly to read the riddle of life. He looked for the most precise and certain of truths, and found them in geometry, as he thought; he strained his eyes to find in them an ethical teaching to guide him "o'er moor and fen, o'er crag and torrent." There is much pathos in these barren attempts to wring a moral significance out of circles and perpendiculars and numbers.

Far more largely, however, the story of Euclid coincides with the record of the fortunes of science generally, until modern times. Like the rest of science, geometry rose and flourished in Greek soil, and faded awhile in that extra-ordinary and universal blight of thought which the Middle Ages witnessed, when individual opinion was lost and truth came on authority. In things temporal, the feudal system ground man down into a serf; in things spiritual, the ecclesiastical system made that serfdom complete. It is not surprising to see a sign of these black times in the fact that proofs were omitted in the mutilated Euclids which were then current. When what was best in civilization had well-nigh deserted Europe, Euclid, along with all true culture, both of science and of art, found an asylum with an alien race—the Arabs amongst them alone scientific research was unfettered.

Then in due time came the great restoration, a new birth like that proclaimed later with the silver trumpet of a Shelley-

> The world's great age begins anew,
> The golden years return,
> The earth doth, like a snake, renew
> Her winter weeds outworn.

The story of Euclid illustrates clearly the intellectual side of this renewal of life, and casts some light even on its religious side, the so called Reformation, in the conflict of Jesuit and Humanist. With the Renaissance is discerned awakening once more the pure love of knowledge, and that passion for the truth which impels research into the new and criticism of the old. Then, to its shame, the European world had to look to the Arabs for its science; it found its Euclid in Spain, and the first of innumerable printed editions was derived from an Arabic text. "In the year of salvation, 1482," the prolific force of the printing-press made copies of the *Elements* obtainable throughout Western Europe.

Englishmen were neither last nor least in this new striving after exact knowledge and independent thought, and in particular the study of Euclid underwent a vigorous revival, in which others beside professed scholars had a share. Thus the first English printed edition of the *Elements* was the work of a "citizen of London," Henry Billingsley, who modestly apologizes to the learned Universities for trespassing on their preserves; and an eloquent introduction from the pen of John Dee, "an old forworne mathematician," addresses the work, not to "Universitie scholers," but to "the handlyng of unlatined people." This splendid volume, of nearly five hundred folio pages, bears the date 1570; and both earlier and later Englishmen did yeoman service for the better understanding of Euclid.

In the last two centuries most branches of science have undergone revolutionary developments, and geometry is no exception to the rule, though the lines of advance are less

widely known. Until lately there would seem to have existed a prevailing impression that geometry, so far as Euclid had formulated it, was stereotyped once for all in the Elements, in a position beyond cavil and criticism. In point of fact, the absolute truth of nearly all Euclid's system is in question; and the concluding chapters of this story will form a commentary on this indisputable if startling statement. There it will be seen that, whereas at the outset geometry is reported to have concerned herself with the measurement of muddy land, she now handles celestial as well as terrestrial problems: she has extended her domain to the furthest bounds of space.

Though the abstractness of geometry may be repugnant to "the natural heart of man," the adoption of the historical method conjures up personalities, and not propositions only. The history of geometry is, as it were, a drama enacted through the centuries therein is beheld the growth of a science from helpless birth and stumbling childhood, through the strenuous but rash vigour of youth, to the perfect wisdom and disciplined strength of manhood. In the life of a science a human life is but an insignificant moment, and from this point of view the history of geometry assumes a grandeur scarcely short of majestic.

CHAPTER II

GEOMETRY FROM BIRTH TO YOUTH

(General Survey, 600-300 B.C.)

The earth beareth fruit of herself; first the blade, then the ear, then the full corn in the ear. - THE GOSPEL.

FROM the third century before the beginning of the Christian era and to a further stage beyond, a single University was mistress of the scientific studies of the world. This queenly University had her seat in the wealthy and populous seaport founded in the Delta of the Nile by Alexander the Great. The city, called Alexandria after its founder, speedily outdistanced all rivals in splendour and opulence, and its University was its proudest boast. The enormous Library, the extensive laboratories and the museums, were thronged by students from all countries, who listened to lectures on philosophy and science.

The chair of geometry was occupied first perhaps by the ablest teacher the University ever had the good fortune to possess: certainly no other teacher has produced a text-book of such lasting merit. The man and the book are, of course, Euclid and his *Elements*, but the book so overshadowed the man that not long after his day Euclid was regularly called the "Elementist." Of his personality little is really known, though some minds may find comfort in the reflections that

he is not a myth, that the question of a Proto-Euclid or of a Deutero-Euclid is easily settled, and that no great space needs to be devoted to a Pseudo-Euclid, or any troublesome "other man of the same name."

Euclid may be seen delivering professorial lectures on geometry to Alexandrian students, freshmen and others and very probably these lectures are the basis of a work which he allowed to be copied and circulated under the title *Elements of Geometry.* Others had anticipated him in the composition of such works, and he would be sure to consult some of these earlier treatises; perhaps they would be contained in the nucleus of the Library. Thus the inquiry is started: How much did Euclid borrow or rearrange, and how much did he himself contribute? And it will be of interest to learn who were the pioneers whose work was used, and how they too acquired their knowledge. The settlement of these points means an investigation of the rise and progress of geometry before Euclid's day, which will be undertaken in succeeding chapters, whilst the present is devoted to a general sketch.

All traditions unite in declaring that the birth of geometry took place in Egypt. Its prime cause was understood to consist in the practical necessity of surveying lands flooded by the Nile, and this is told in his quaint way by John Dee, writing some years before the time of the Armada. He says:

> "This science of magnitude, his properties, conditions, and appurtenances, commonly now is, and from the beginning hath of all philosophers been, called Geometry. But, verily, with a name too base and scant for a science of such dignity and ampleness. And perchance that name, by common and secret

consent of all wise men, hitherto hath been suffered to remain, that it might carry with it a perpetual memory of the first and notablest benefit by that science to common people showed:

"Which was, when bounds and meres of land and ground were lost and confounded, as in Egypt yearly with the overflowing of Nilus, the greatest and longest river in the world, upon these and such like occasions, some by ignorance, some by negligence, some by fraud, and some by violence, did wrongfully limit, measure, encroach, or challenge, by pretence of just content and measure, those lands and grounds; and so great loss, disquietness, murder, and war did full often sue, till by God's mercy and man's industry the perfect science of lines, planes, and solids, like a divine justiciar, gave unto every man his own.

"The people then by this art pleasured, and greatly relieved in their land's just measuring; and other philosophers writing rules for land-measuring; between them both thus confirmed the name of Geometry, that is, according to the very etymology of the word, Land-measuring."

To the Egyptians is thus assigned the credit of the beginnings of geometry some unknown number of centuries before Christ; and at the least it is certain that the Greeks took the cue from them. Through imposing a narrow meaning on the word "useful," the Egyptian mind advanced no further than a few rules of thumb. From their short book of mensuration the Greeks extracted a page, and copied it but the copy was infinitely more valuable than the original. The agile and energetic Greek temperament, devoted to science for its own sake, needed no more than a beckoning hand towards realms for conquest.

To be just, the Egyptians were able to effect a good deal beside this reported recovery of boundaries obliterated by

mud: they had geometrical rules applicable to religious ends. Immemorial temples are found to have had their sites pitched with an exceptional care, so that their situation with reference to the heavenly bodies might be true; and so, in the ecclesiastical, as well as in the civil world, the Egyptians had a fund of "useful" geometry. But in this confinement of scope to the "useful" all hope of progress died; and, for all they cared, knowledge might have continued in the same state for another thousand years.

True Geometry was begotten when the Greek thinker, Thales, left his home on the shores of Asia Minor, and travelled far and wide, partly for the sake of commerce, but more from a restlessness which was the sign of awakening life. It is twenty-five centuries since Thales visited Egypt, and then, as now, that mysterious country exercised all the fascinations of an inscrutable antiquity. Above all, its priests, charged with the cherished and hidden traditions of ages, must have excited the admiration of the philosopher to him they would represent the highest culture of an ancient nation; and from them he learned their rough-and-ready geometrical methods which, on his return, he investigated in a more abstract manner. The discoveries thus made will be detailed later: here may be simply recorded the titles bestowed on him by his compatriots: The Ancient, The Sage, The Father of Philosophy. Thales is not only the father of geometry, but of all science.

Since it is proposed at present only to sketch briefly the progress of geometry before Euclid, only the most brilliant of the Elementist's forerunners will be mentioned. Among these, Pythagoras belonged to the generation following Thales.

Gathering about himself a band of disciples, he founded a veritable college, endowed with much genius and little money. His personality is more shadowy than might otherwise have been the case, because of a peculiar custom of his followers. For long after his death they formed an exclusive sect, with "plain living and high thinking" for their rule of life. Pushing humility and loyalty to the extreme, they ascribed all their discoveries to "the Master," as they |were wont to call him. The formula, the Master said, has become a proverb, and it is impossible not to admire such devotion, though it hides the real Pythagoras from modern eyes.

The best known among the names of the other great pioneers is that of Plato, but it must not be forgotten that his fame was not earned in geometrical fields. The "Divine Philosopher" received a mathematical education of the highest excellence, such as might be balanced today by courses at English, German, and French Universities. His keenness for geometry was great, and his enthusiasm lasting; indeed, his admiration for geometrical studies is shown by the warning placed by him over the door of his Academy[1]

No entrance to the ungeometrical.

In his ideal Republic, geometry is to receive the attention it deserves, because it is "the knowledge of that which is everlasting." Hereon John Dee soliloquizes in words which are peculiarly impressive in their eloquence:

1 The building in which he lectured at Athens, about 400 B.C., and which has given its name to similar institutions.

> This was Divine Plato his judgment, both of the purposed, chief, and perfect use of geometry, and of his second depending derivative commodities. And for us Christian men a thousand, thousand more occasions are to have need of the help of megathological contemplations, whereby to train our imaginations and minds, by little and little, to forsake and abandon the gross and corruptible objects of our outward senses, and to apprehend by sure doctrine demonstrative, things mathematical.

The fourth century before the era of Grace was rich in talented geometers, all of Greek blood, who pushed geometrical knowledge into regions far beyond the range of the *Elements*. In fact, Euclid's book was devised for beginners, and contained but a tithe of the geometry known to the great teacher: it would comprise only as much as he judged appropriate to the end he had in view, that is, to furnish an introduction to geometry, and to attain a certain goal in his last book.

Such is in brief the course of geometrical discovery up to Euclid's day. In the three succeeding chapters will be examined more leisurely what has been rapidly reviewed, but two facts call for more immediate remark.

First, the science of geometry did not spring into existence full-grown from the brain of Euclid, as the goddess Minerva was fabled to have emerged in perfect womanly form from the head of Jupiter. The *Elements* were in germinal life centuries before Euclid's birth, when the seed was transplanted by Thales from Egypt. It was then an apparently weakly and insignificant thing, but planted in Greek soil it sprouted, and in the time of the Pythagorean Brotherhood came to have a beautiful if diminutive form. After their day it passed by successive gradations into the full-grown tree, the sturdy and

shapely oak which Euclid revealed to the world. Geometrical history is incomprehensible apart from evolutionary principles.

Secondly, the legion of lost works on geometry, both prior and subsequent to Euclid's day, are witnesses to the survival of the fittest. Euclid took from earlier works what he thought good to repeat and perpetuate. Then these earlier works died that the *Elements* might live, and gained no more than that shadowy kind of immortality described by a German novelist,[2] an immortality resting on the fact that a person's ideas live on more or less in the minds of generations to come. On the other hand, later works did not achieve even this much: unable successfully to combat the *Elements*, they went down under Euclid's victorious chariot-wheels.

Although it was principally in revision and arrangement that Euclid's work consisted, the chiefest mathematicians since the composition of the *Elements* have seen in them the working of a supremely masterly mind.

2 Gustav Freytag, in his "Lost Manuscript: " but the view has not been without lofty and generous exposition in the world of English literature.

CHAPTER III

THE FATHER OF GEOMETRY

(660-550 B.C.)

He was happy, if to know
Causes of things be happiness.
M. ARNOLD.

IT is now time to proceed to a closer view of the pioneers who lived and worked for the furtherance of geometrical truth in the three centuries before Euclid, and the present chapter will be devoted to the first of them—Thales, the founder of geometry.

Born at the dawn of European civilization, when its earliest rays were coming from the East, Thales was a native of Miletus, a flourishing Greek city on the Western shores of Asia Minor, between which and Greece proper lay—

The Isles of Greece,
Where burning Sappho loved and sung,
Where grew the arts of war and peace.

He soon became famous for a knowledge and wisdom which gave him rank among the Seven Sages, and of his acumen and enterprise many fireside tales were told.

One of these stories speaks of a mule possessed of the undesirable habit of lying down in the stream when a ford

was crossed. As the burden was composed of salt, the animal's behaviour did some credit to its reasoning powers, and may not have been entirely due to perversity. But Thales was astute enough to effect a complete cure by loading the mule for several journeys with nothing but sponges. He is also reported once to have made a corner in olives by buying up a whole year's produce, which he then sold at his own price, thus inaugurating, if the tale is not idle, one of the nefarious practices of modern commerce.

During the earlier part of a long life, Thales visited Egypt, where his passion for knowledge exercised itself in such wise that he quickly exhausted the slender stock of mensuration in the possession of the Egyptian priests. Six hundred years before Christ is a not improbable date to assign to the visit, and it may be recollected that a little previously Egypt was still a closed empire, like the China of a few years ago. The sphinx-like character of the nation, its antiquity and mystery, impressed the Greek visitor and priests of a cult which extended back for uncounted generations could patronize their guests and declare, "You are but children." Children were Thales and his contemporaries, indeed, but they grew fast, and soon outstripped a civilization that had reached its dotage.

Lower Egypt was, and is, practically rainless. Only the irrigation of the Nile in its annual flood kept back the encroaching desert-sand: but for its regular overflow desolation was doomed to reign supreme over a land smiling with crops and bedecked with dwellings.

Year by year Father Nile overflowed his banks, and fertilized with rich mud fields far and wide. To know the time of this

visitation, and to retrace the lost boundaries, [3] were problems for the ruling class, and which some time or other had been faced successfully by the priests. It would be easy to exaggerate the extent of the priestly code of learning, since the thicker the mist, the greater do things loom; the high-water-mark of their geometry was doubtless attained in their knowledge that—

If the sides of a triangle measure 3, 4 and 5, then the greatest angle is a right angle.

This fact was of use to them in the erection of temples facing the proper point of the heavens. Above, they saw moving serenely and resolutely across the sky the bright emblems of a Power demanding their adoration; and towards these their sacred buildings looked at fitting seasons of the year. In plotting out such sites, cords of lengths as specified above were pegged out on level ground, and the right angle so constructed afforded them much assistance. Why the angle should be right, the Egyptian mind very probably neither knew nor cared.

This, and much like it, Thales the Wise would learn with avidity, though the materialism and guesswork of the Egyptians might leave him extremely dissatisfied. In his mind the Egyptian cords were refined until they became Greek lines: the place of the concrete cord was taken by the abstract idea of length without breadth or thickness, and thus may have been born the notion of a "line," now so familiar and then so strange.

3 De Morgan chose to scoff at this tradition of the beginning of geometry. Calling it the "stock" history, he quoted the couplet-

To teach weak mortals property to scan
Down came Geometry, and formed a plan!

Things went pretty much the same way in the science of astronomy. The Egyptians observed eclipses as they came round annually, made notes of the occasions, and registered them. Another Oriental nation, the Chaldeans, kept like records, though with greater fruit in their discovery of the extraordinary fact that eclipses repeat themselves after an interval of eighteen years plus eleven days.[4] But it was left to Thales and Western minds to pierce below appearances: Thales sowed the seed of which Galileo and Newton gathered the increase.

The mensuration which he learned from the Egyptians, Thales purified, making out of their practice his theory. To them he would seem a dreamer, an academician remote from the realities of a busy pushing world. Their desire did not extend beyond that knowledge of which they clearly perceived the proximate utility; he appears rather to have loved knowledge for its own sake, apart from the good things to be immediately and obviously gained from it. And yet (gain coming, as ever, by loss) Thales has proved a greater benefactor of the entire human family than all the Egyptian priests with all their practical rules.

From Egypt, Thales returned to his native city, and there enjoyed the calm of the study after the bustle of the market. Yet so comprehensive was his genius that to the end of his life his interest in public affairs was maintained. Meanwhile a little band of eager students attached themselves to "the Father of Geometry," and after his death, not far from 550 B.C., his teaching lived on in the school he had thus founded.

In one sense, the contributions to geometry made by Thales

4 In this way future eclipses were predicted from a knowledge of those past.